XIX JUNIOR BALKAN MATHEMATICAL OLYMPIAD
Belgrade, Serbia

19th Junior Balkan Mathematical Olympiad
June 24-29, 2015, Belgrade, Serbia

Language: *English*
Friday, June 26, 2015.

1. Find all prime numbers a, b, c and positive integers k satisfying the equation

$$a^2 + b^2 + 16c^2 = 9k^2 + 1.$$

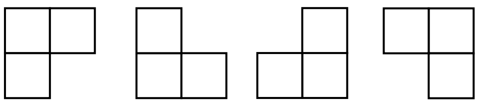
2. Let a, b, c be positive real numbers such that $a + b + c = 3$. Find the minimum value of the expression

$$A = \frac{2-a^3}{a} + \frac{2-b^3}{b} + \frac{2-c^3}{c}$$

3. Let ABC be an acute triangle. The lines l_1 and l_2 are perpendicular to AB at the points A and B, respectively. The perpendicular lines from the midpoint M of AB to the lines AC and BC intersect l_1 and l_2 at the points E and F, respectively. If D is the intersection point of the lines EF and MC, prove that

$$\angle ADB = \angle EMF.$$

4. An L-shape is one of the following four pieces, each consisting of three unit squares:



A 5×5 board, consisting of 25 unit squares, a positive integer $k \leq 25$ and an unlimited supply of L-shapes are given. Two players, A and B, play the following game: starting with A they alternatively mark a previously unmarked unit square until they mark a total of k unit squares.

We say that a placement of L-shapes on unmarked unit squares is called *good* if the L-shapes do not overlap and each of them covers exactly three unmarked unit squares of the board.

B wins if every *good* placement of L-shapes leaves uncovered at least three unmarked unit squares. Determine the minimum value of k for which B has a winning strategy.

Time: 4 hours and 30 minutes
Each problem is worth 10 points